

Behavior of Eccentrically Loaded Plain and Steel Fiber Reinforced Concrete Filled Steel Box Columns

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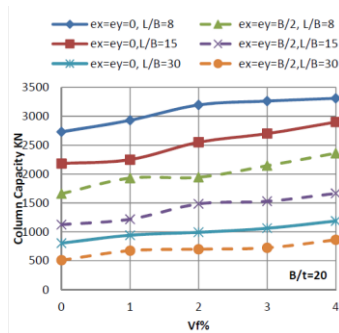
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Graphical abstract



Abstract

The present study investigates the behavior of steel fiber reinforced concrete filled steel box columns (SFRCFSBC) targeting to enhance their strength. A nonlinear finite element model using ANSYS program has been developed to investigate the structural behavior of the inspected columns. The results obtained from that model have been compared with those calculated using Euro code (EC4), AISC/LRFD (2005) and the Egyptian Code of Practice for Steel Construction (ECPSC/LRFD 2007). The comparison indicated that the results of the model have been evaluated to an acceptable limit of accuracy. A parametric study was carried out to investigate the effect of wall thickness, column slenderness and percentage of steel fiber in concrete on the ultimate strength of composite columns. Confinement of the concrete core provided by the steel case was also investigated. It can be concluded from the results that a considerable increase in compressive and flexural strength may be gained by increasing the steel fiber percentage up to 4%. The highest rate of increase in strength for long columns was about 20% by using steel fiber percentage between 0.5% and 1.0%, while for short and medium columns was about 10% by using steel fiber percentage between 1% and 2%.

Keywords: Composite columns; steel tube; finite element; steel fibers; slenderness ratio

Abstrak

Kajian ini menyiasat tingkah laku tiang kotak keluli terisi konkrit bergentian keluli diperkukuh (SFRCFSBC) dengan menyasarkan untuk meningkatkan kekuatan elemen ini. Satu model unsur terhingga tidak linear menggunakan program ANSYS telah dibangunkan untuk menyiasat tingkah laku struktur ruang yang diperiksa. Keputusan yang diperolehi telah dibandingkan dengan keputusan yang berdasarkan kod Euro (EC4), AISC / LRFD (2005) dan Kod Amalan Pembinaan Keluli Mesir (ECPSC / LRFD 2007). Perbandingan menunjukkan bahawa keputusan pemodelan telah dinilai pada tahap ketepatan yang boleh diterima. Kajian parametrik telah dijalankan untuk mengkaji kesan ketebalan dinding, kelangsingan tiang dan peratusan gentian keluli dalam konkrit pada kekuatan muktamad tiang komposit. Perlindungan ke atas teras konkrit oleh bekas keluli juga disiasat. Kesimpulan berdasarkan hasil dapatan menunjukkan yang peningkatan ketara dalam kekuatan mampatan dan lenturan boleh diperolehi dengan meningkatkan peratusan gentian keluli sehingga 4%. Kadar tertinggi peningkatan dalam kekuatan untuk tiang yang panjang adalah kira-kira 20% dengan menggunakan peratusan gentian keluli antara 0.5% dan 1.0%, manakala bagi tiang pendek dan sederhana peningkatannya adalah kira-kira 10% dengan menggunakan peratusan gentian keluli antara 1% dan 2%.

Kata kunci: Tiang komposit; tiub keluli; unsur terhingga; nisbah kelangsingan

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1.0 INTRODUCTION

Concrete-filled steel tube columns have been extensively used in modern structures due to the advantages of the steel tube and concrete core. Many kinds of in-fill materials are used to improve strength behavior of composite columns. Among the various in-fill materials, steel fiber is gaining attention due to high flexural strength, tensile strength, lower shrinkage, and better fire resistance. The characteristics of fiber reinforced concrete change

with varying fiber materials, geometries, distribution, orientation and densities. The main objective of this research is to investigate the behavior and properties of steel fiber reinforced concrete-filled steel box columns (SFRCFSBC). A developed finite element program using ANSYS software is used in the analysis. The material nonlinearities of concrete and high strength steel box as well as concrete confinement were considered in the analysis. Parametric study is conducted to investigate the effects of slenderness ratio L/B , wall thickness of steel box and concrete

strength on the behavior and strength of the steel fiber reinforced concrete filled steel box columns. The results obtained from the finite element model has been compared with those obtained from a recent experimental work made by Mursi, M. and Uy, B. (2003) [1], Mursi, M. and Uy, B. (2004) [2], and Schneider (1998) [3], as well as the results obtained using Euro-code (EC4)[4], AISC/LRFD (2005) [5] and Egyptian code of practice for steel construction ECPSC/LRFD (2007) [6].

2.0 FINITE ELEMENT MODEL

2.1 General

The physical behavior of steel fiber reinforced concrete-filled steel box columns is simulated by modeling the components of these columns. These components are; (a) the confined concrete containing steel fibers, (b) the steel box, (c) the steel plates as a loading jacks and (d) the interface between the concrete and the steel box. In addition to these parameters, the choice of the element type and mesh size that provide reliable results with reasonable computational time is also important in simulating structures with interface elements, Abdullah, S. (2012) [7].

2.2 Finite Element Type and Mesh

The concrete core of fiber reinforced concrete filled steel box columns is modeled using 8-node brick elements, with three translation degrees of freedom at each node (element; SOLID 65 in ANSYS12.0) [8]. Steel fibers is modeled in concrete using the rebar option included in SOLID 65 real constant by defining the steel fiber material properties, volumetric ratio and orientation angle in x, y and z directions. The steel box is modeled using a 4-node shell element, with six degrees of freedom at each node (element; SHELL 63 in ANSYS12.0) [8]. Inelastic material and geometric nonlinear behavior are used for this element. Von Mises yield criteria is used to define the yield surface. No strain-hardening is assumed for the steel box. Thus, if strain-hardening characteristics are observed in concrete filled steel box column behavior, it is primarily due to the interaction between the steel and concrete components. A 50 mm thick steel plate, modeled using (element; SOLID 45 in ANSYS12.0) [8], was added at the support locations in order to avoid stress concentration problems and to prevent localized crushing of concrete elements near the supporting points and load application locations. The gap element is used for the interface between the concrete and the steel components. The gap element has two faces; when the faces are in contact; compressive forces develop between the two materials resulting in frictional forces. The friction coefficient used in the analysis is 0.25. On the other hand, if the gap element is in tension, the two faces become separated from each other, resulting in no contact between the concrete and steel, and consequently no bond is developed. TARGE170 is used to represent various 3-D “target” surfaces for the associated contact elements (CONTA173). Figure 1 shows the finite element mesh of the concrete-filled steel box column.

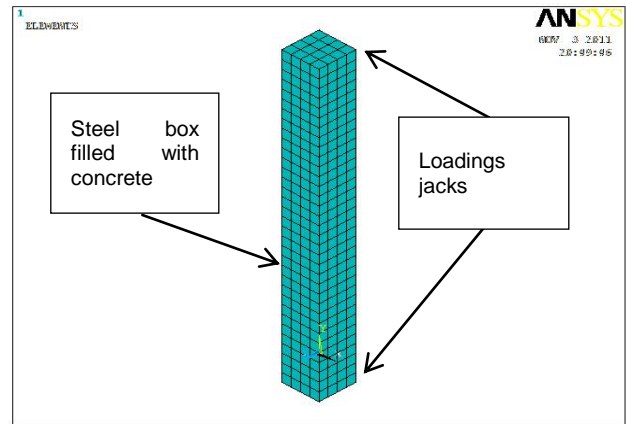


Figure 1 Typical model of concrete filled steel box columns

2.3 Boundary Condition and Load Application

The top surface of the column is prevented from displacement in the X and Z directions but allows displacement to take place in the Y direction. On the other hand, the bottom surface of the column is prevented from displacement in the X and Z directions and prevented from displacement in Y direction at the point opposite to the point of load application at the top of column. The corners of the steel tube are assumed to be exactly 90° and corner radii are not considered. The compressive load is applied to the top surface in the Y direction through a rigid steel cap to distribute the load uniformly over the cross section.

2.4 Material Modeling of Steel Box

The uni-axial behavior of the steel box can be simulated by an elastic-perfectly plastic model as shown in Figure 2. When the stress points fall inside the yield surface, the steel box behavior is linearly elastic. If the stresses of the steel box reach the yield surface, the steel box behavior becomes perfectly plastic. Consequently, the steel tube is assumed to fail and becomes unable to resist any further loading.

In the analysis, the Poisson's ratio ν_s of the steel tube is assumed to be $\nu_s = 0.3$, the modulus of elasticity $E_s = 210000$ MPa, yield stress $f_y = 360$ MPa.

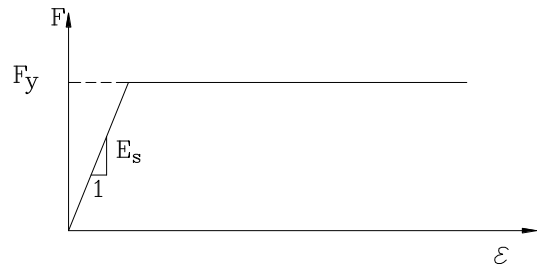


Figure 2 Elastic perfectly plastic model for steel box

2.5 Material Modeling of Steel Fiber Reinforced Concrete Core

Equivalent uni-axial stress–strain curves for both unconfined and confined concrete are shown in Figure 3, where f_c is the unconfined concrete cylinder compressive strength, which is

equal to 0.8 (f_{cu}), and (f_{cu}) is the unconfined concrete cube compressive strength. The corresponding unconfined strain (ϵ_c) is taken as 0.003. The confined concrete compressive strength (f_{cc}) and the corresponding confined strain (ϵ_{cc}) can be determined from (1) and (2), respectively, proposed by Mander *et al.* (1988) [9].

$$f_{cc} = f_c + k_1 f_l \quad (1)$$

$$\epsilon_{cc} = \epsilon_c \left(1 + k_2 \frac{f_l}{f_c} \right) \quad (2)$$

Where f_l , is the lateral confining pressure imposed by the steel box. The lateral confining pressure (f_l) depends on the B/t ratio and the steel tube yield stress. The approximate value of (f_l) can be obtained from empirical equations given by Hu *et al.* (2003) [10], where a wide range of B/t ratios ranging from 17 to 150 are investigated. The value of (f_l) has a significant effect for steel tubes with a small B/t ratio. On the other hand, the value of (f_l) is equal to zero for steel tubes with B/t ratios greater than or equal to 29.2.

$$f_l/f_y = 0.055048 - 0.001885(B/t) \quad (17 \leq B/t \leq 29.2)$$

$$f_l/f_y = 0 \quad (29.2 \leq B/t \leq 150)$$

The factors (k_1) and (k_2) are taken as 4.1 and 20.5, respectively, as given by Richart *et al.* (1928) [11].

To define the full equivalent uni-axial stress–strain curve for confined concrete as shown in Figure 3, three parts of the curve have to be identified.

The first part is the initially assumed elastic range to the proportional limit stress. The value of the proportional limit stress is taken as $0.5(f_{cc})$ as given by Hu *et al.* (2003) [11]. The initial Young's modulus of confined concrete (E_{cc}) is reasonably calculated using the empirical equation (3) given by ACI (1999) [12]. The Poisson's ratio (ν_{cc}) of confined concrete is taken as 0.2.

$$E_{cc} = 4700\sqrt{f_{cc}} \text{ MPa} \quad (3)$$

The second part of the curve is the nonlinear portion starting from the proportional limit stress $0.5(f_{cc})$ to the confined concrete strength (f_{cc}). This part of the curve can be determined from (4), which is a common equation proposed by Saenz (1964) [13]. This equation is used to represent the multi-dimensional stress and strain values for the equivalent uniaxial stress and strain values. The unknowns of the equation are the uni-axial stress (f) and strain (ϵ) values defining this part of the curve. The strain values (ϵ) are taken between the proportional strain, which is equal to $(0.5 f_{cc}/E_{cc})$, and the confined strain (ϵ_{cc}), which corresponds to the confined concrete strength. The stress values (f) can be determined easily from (4) by assuming the strain values (ϵ).

$$f = \frac{E_{cc} \epsilon}{1 + (R + R_E - 2) \left(\frac{\epsilon}{\epsilon_{cc}} \right) - (2R - 1) \left(\frac{\epsilon}{\epsilon_{cc}} \right)^2 + R \left(\frac{\epsilon}{\epsilon_{cc}} \right)^3} \quad (4)$$

Where R_E and R values are calculated from (5) and (6) respectively:

$$R_E = \frac{E_{cc} \epsilon_{cc}}{f_{cc}} \quad (5)$$

$$R = \frac{R_E (R_E - 1)}{(R_E - 1)^2} - \frac{1}{R_E} \quad (6)$$

While the constants R_σ and R_ϵ are taken equal to 4.0, as recommended by Hu and Schnobrich (1989) [14].

The third part of the confined concrete stress–strain curve is the descending part used to model the softening behavior of concrete from the confined concrete strength f_{cc} to a value lower than or equal to $K3 f_{cc}$ with the corresponding strain of $11\epsilon_{cc}$. The reduction factor ($k3$) depends on the B/t ratio and the steel tube yield stress (f_y). The approximate value of $k3$ can be calculated from empirical equations given by Hu *et al.* (2003) [10].

$$k3 = 0.000178(B/t)^2 - 0.02492(B/t) + 1.2722 \quad (17 \leq B/t \leq 70)$$

$$k3 = 0.4 \quad (70 \leq B/t \leq 150)$$

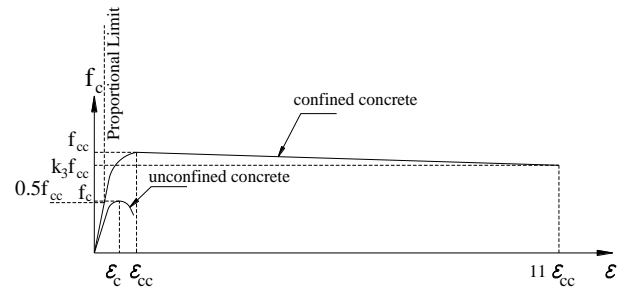


Figure 3 Equivalent uniaxial stress–strain curves for confined and unconfined concrete

3.0 VERIFICATION OF FINITE ELEMENT MODEL

Steel box columns from Mursi, M. and Uy, B. (2003) [1], four concrete filled steel box columns from Mursi, M. and Uy, B. (2004) [2] and three concrete filled steel box columns from Schnieder, S.P. (1998) [3] are used to verify the proposed finite element model for concrete filled steel box columns.

Table 1 lists the dimensions, B/t, L/B ratios, and material properties of the analyzed concrete filled steel box columns.

The results of concentric and eccentric capacities of the concrete filled steel box columns using the suggested finite element model, Nmodel are compared with the experimental results given by Mursi, M. and Uy, B. (2003)[1], Mursi, M. and Uy, B. (2004) [2] and Schnieder, S.P. (1998) [3], Nexp.

The analytical results are compared with the design equations of the AISC/LRFD (2005) [5] Specification, N_{AISC} , and the ones by the Egyptian code of practice for steel construction [6], N_{ECPSC} , and the results by Euro code 4[4], N_{EC4} are listed in table 2.

From table 2, it can be noticed that: The model's behavior is of accepted compliance compared to the experimental results. The comparison indicates that the proposed finite element model provides very close estimates for determining the axial capacities of concrete filled steel box columns compared to the three design codes.

Table 1 Geometry and material properties of concrete filled steel box columns

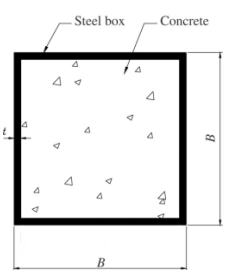
Column Name	B (mm)	t (mm)	Length "L" (mm)	B/t	L/B	f_y (MPa)	F_c (MPa)	Reference	Cross section
C-X-S-40	104	3	2800	34.7	26.92	269	65	"Mursi, M. and Uy, B. (2003)"	
C-X-S-50	134	3	2800	44.7	20.89	269	65		
C-X-S-60	164	3	2800	54.7	17.07	269	65		
SH-C210S	210	5	730	42	3.48	761	20	"Mursi, M. and Uy, B. (2004)"	
SH-C260S	260	5	880	52	3.38	761	20		
SH-C210L	210	5	3020	42	14.38	761	20		
SH-C260L	260	5	3020	52	11.62	761	20	"Schnieder, S.P. (1998)"	
S1	127	3.15	610	40.4	4.8	356	30.454		
S2	127	4.34	610	29.2	4.8	357	26.044		
S3	127	4.55	610	22.3	4.8	322	23.805		

Table 2 Comparison between the finite element model outputs and corresponding results obtained from experimental studies, nominal EC4, AISC/LRFD and ECPSC/LRFD specifications

Column Name	e (mm)	e/L	Results				Comparison				
			N_{exp} (kN)	N_{EC4} (kN)	N_{AISC} (kN)	N_{ECPSC} (kN)	N_{model} (kN)	$\frac{N_{model}}{N_{EC4}}$	$\frac{N_{model}}{N_{AISC}}$	$\frac{N_{model}}{N_{ECPSC}}$	$\frac{N_{model}}{N_{exp}}$
C-X-S-40	10	0.096	736	633	573	-*	739	1.167	1.289	-*	1.004
C-X-S-50	15	0.112	1090	1036	1023	-*	1068	1.03	1.044	-*	0.979
C-X-S-60	20	0.128	1444	1357	1579	-*	1421	1.047	1.047	-*	0.984
SH-C210S	0	0	3609	3503	3762	3769	3584	1.023	0.953	0.951	0.993
SH-C260S	0	0	3950	5024	4897	4798	3972	0.791	0.811	0.828	1.006
SH-C210L	20	0.95	2939	2960	3200	3267	3000	1.014	0.938	0.918	1.023
SH-C260L	25	0.96	3062	3883	4431	4376	3080	0.793	0.695	0.704	1.006
S1	0	0	917	992	852	916	904	0.911	1.061	0.987	0.986
S2	0	0	1095	1064	1058	1058	1158	1.088	1.095	1.095	1.058
S3	0	0	1113	993	988	990	1040	1.047	1.053	1.051	0.934
Mean								0.991	0.998	0.933	0.997
Standard Deviation								0.123	0.163	0.134	0.032

*Note: The ECPSC recommends that f_c should not be taken more than 50 MPa.

4.0 PARAMETRIC STUDY

A parametric study is conducted using the proposed model on various steel fiber reinforced concrete filled steel box columns to investigate the effect of four main parameters on the ultimate capacities of SFRCFSC. These parameters are: the column width to wall thickness ratio, B/t; the slenderness ratio, L/B; the percentage of steel fibers in concrete, V_f %; and the load eccentricity effect.

4.1 Column Width To Wall Thickness Ratio

The first parameter, B/t, has a great effect on concrete confinement. Concrete filled steel box columns with high value

of B/t ratio provide poor confinement for concrete. A premature failure of the composite column due to steel box local buckling will result in inadequate confinement for concrete. Increasing wall thickness and reducing B/t ratio provide remarkable confinement for concrete

4.2 Slenderness Ratio

The effect of column slenderness ratio L/B, on the ultimate strength capacity of SFRCFSC is investigated. In this case, three L/B ratios equal to 8, 15, and 30 are considered for short, medium and long respectively for the different B/t ratios.

4.3 Percentage of Steel Fiber in Concrete V_f %

The third parameter concerns with the effect of steel fiber percentage in concrete V_f %. The steel fiber percentage in concrete is taken equal to 0% for plain concrete and up to 4%.

4.4 Effect of Load Application

The last parameter discusses the effect of loading application. The compression loads are considered centric and eccentric. The eccentricity effect (e_x/B and e_y/B) is considered to be equal to 0.5.

The geometry and material properties of the analyzed steel fiber reinforced concrete filled steel box columns are illustrated in Table 3.

Table 3 Geometry and material properties of concrete filled steel box columns

Columns	Dimensions (mm)			L/B ratio	B/t ratio	Concrete Strength		Steel yield strength
						Unconfined	Confined	
	B	t	L			f_c	f_{cc}	
C01			1600	8				
C02	200	10	3000	15	20	30	55.6	360
C03			6000	30				
C04			1600	8				
C05	200	8	3000	15	25	30	41.7	360
C06			6000	30				
C07			1600	8				
C08	200	5	3000	15	40	30	30	360
C09			6000	30				

5.0 DISCUSSION OF THE RESULTS

5.1 Effect of Steel Plate Wall Thickness

Wall thickness of steel box has a great effect on short to medium height columns. Compact steel plate for column wall thickness provides more confinement to the concrete core that contributes to an increase in the overall column capacity. Non-compact steel plate for wall column thickness provides less confinement that will result in substantial decrease in the column's capacity. The effect of column wall thickness on long columns has a less effect on the column behavior due to the overall column buckling. Figure 4 indicates the axial and eccentric capacity versus B/t ratio for different L/B ratios.

5.2 Effect of Column Slenderness Ratio

The increase in column's height has a minor effect on short and medium columns, the fail is attributed to the inelastic buckling, meaning that, the column fails by the crushing of concrete and/or yielding of the steel plates then the column capacity decreases in percentage ranges from 3 % to 30 %, while for long columns, the column's height has a great influence on the column's capacity that fails due to overall buckling before the crushing of concrete and/or yielding of the column steel plates. Figure 5 shows the axial and eccentric column's capacity versus

L/B ratio for different B/t ratios, and Figure 6 shows axial column capacity versus L/B ratio for different value of V_f %.

5.3 Effect of Percentage of Steel Fiber

Figure 7 plots the axial and eccentric column capacity versus steel fiber percentage in the in-filled concrete, V_f % for different B/t and L/B ratios. Table 4 presents the percentage of increase in column capacities with the increase in V_f % for the different columns' aspect ratios, under the different axial centric and eccentric load cases.

For short and medium columns, increasing the percentage of fibers from 0% to 4%, will lead to an increase in column's capacity in percentage varies from 3% to 28% for axial load and from 6% to 30% for eccentric load. The highest rate of increase lies within a percentage of fibers between 1% and 2%. Therefore it is recommended to use 1.5% of steel fibers in case of short and medium columns.

For long columns, increasing the percentage of fibers from 0% to 4%, will lead to an increase in column's capacity by percentage varies from 22% to 37% for axial load, and from 16% to 50% for eccentric load. The highest rate of increase lies between 0% and 1%. Therefore it is recommended to use 0.5% of steel fibers in case of long column.

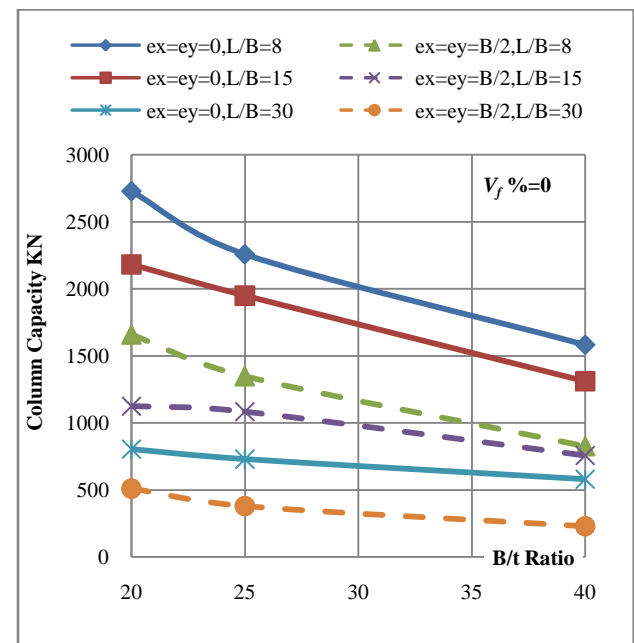


Figure 4 Column capacities versus B/t ratio for different L/B ratios

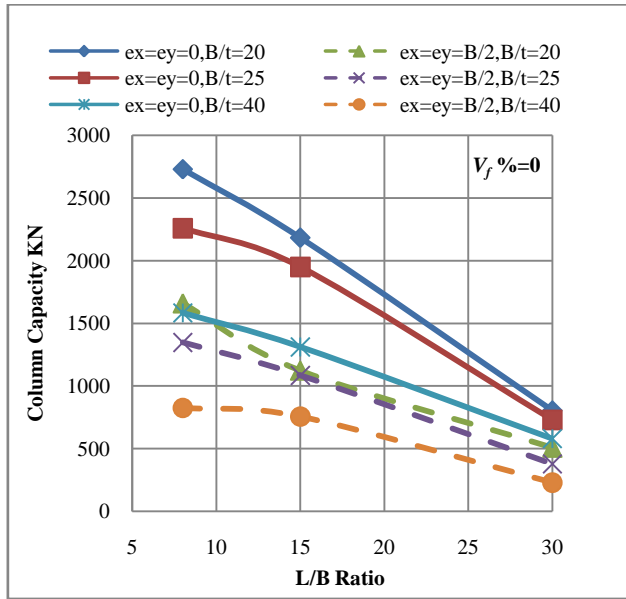


Figure 5 Column capacity versus L/B ratio for different B/t ratios

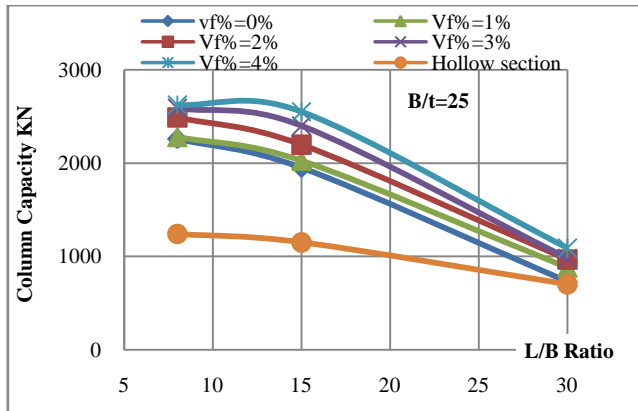


Figure 6 Column capacity versus L/B ratio for different $V_f\%$

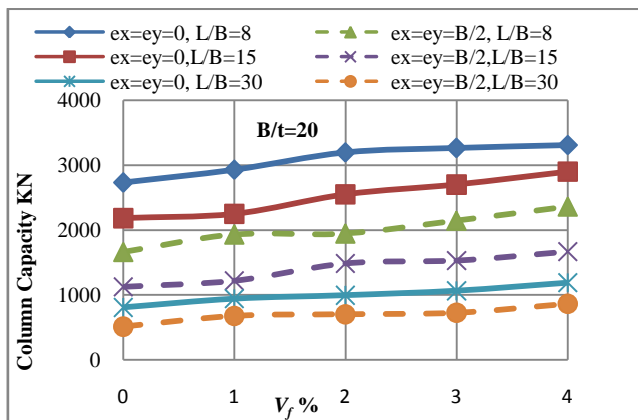


Figure 7 Column capacity versus $V_f\%$ for different L/B ratios

Table 1 Increasing percentage in column strength with variation of steel fiber ratio

Column Type	Load type	Steel fiber percentage $V_f\%$			
		1%	2%	3%	4%
Short column (L/B=8)	Centric Axial load	5	15	19	21
	Axial load with uniaxial moment	6	15	24	27
	Axial load with biaxial moment	12	18	16	31
Medium Column (L/B=15)	Axial load	3	13	23	28
	Axial load with uniaxial moment	8	23	29	34
	Axial load with biaxial moment	8	18	21	28
Long Column (L/B=30)	Axial load	22	27	29	37
	Axial load with uniaxial moment	16	24	31	37
	Axial load with biaxial moment	23	36	43	50

6.0 CONCLUSION

The developed finite element model is used to predict the behavior of fiber reinforced concrete filled steel box columns. From the analysis performed on different sets of Fiber reinforced concrete filled steel box columns, the following conclusions are found:

- 1) The results obtained from the developed model exhibits good correlation with the available experimental results as well as the calculated results applying the Euro code (EC4), AISC/LRFD (2005) and the Egyptian code of practice for steel construction ECPSC/LRFD (2007).
- 2) The ratio of B/t significantly affects the behavior of concrete filled steel box columns. In general, enlarging B/t ratio decreases the confinement effect of steel tube to the in filled concrete for both the axial and eccentric load cases.
- 3) Wall thickness of steel box has a great effect on short to medium columns. Increasing the steel plate thickness results in substantial increase in overall column's capacity, while long columns fail due to the overall column's buckling therefore, increasing the wall thickness has a limited effect.
- 4) The slenderness ratio L/B has a very remarkable effect on the strength and behavior of concrete filled steel box columns under axial and eccentric loading.
- 5) The results show that the axial and eccentric capacities of concrete filled steel box columns increase along with the increase of the concrete strength.
- 6) The use of SFRC has resulted in considerable improvement in the structural behavior of concrete filled steel box columns subjected to axial and eccentric loading.
- 7) For short and medium columns, increasing the percentage of fibers from 0% to 4%, has led to an increase in column's capacities up to 20% and 30% for centric and eccentric loading cases, respectively. The highest rate of increase in column's strength is gained by using steel fiber percentage between 1% and 2%. Therefore it is recommended to use 1.5% of steel fibers that will lead to an increase in column strength up to 10%.

For long columns, increasing the percentage of fibers from 0% to 4% has led to an increase in column's capacity up to 50%. The biggest rate of increase in column strength is gained by using steel fiber percentage between 0% and 1%. Therefore it is recommended to use 0.5% of steel fibers that will lead to an increase in column strength up to 20%.

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